

Physics 110B
Homework Solution #1

#1. (Griffiths 9.11)

$$\begin{aligned}
 \langle fg \rangle &= \frac{1}{T} \int_0^T A \cos(k \cdot r - \omega t + \delta_a) B \cos(k \cdot r - \omega t + \delta_b) dt \\
 &= \frac{AB}{2T} \int_0^T dt (\cos(k \cdot r - \omega t + \delta_a + k \cdot r - \omega t + \delta_b) + \cos(k \cdot r - \omega t + \delta_a - k \cdot r + \omega t - \delta_b)) \\
 &= \frac{AB}{2T} \int_0^T dt (\cos(2k \cdot r - 2\omega t + \delta_a + \delta_b) + \cos(\delta_a - \delta_b)) \\
 &= \frac{AB}{2T} \cos(\delta_a - \delta_b) T = \frac{AB}{2} \cos(\delta_a - \delta_b)
 \end{aligned}$$

Now, in complex notation:

$$\begin{aligned}
 \tilde{f} &= A \exp[i(k \cdot r - \omega t + \delta_a)], \quad \tilde{g} = B \exp[i(k \cdot r - \omega t + \delta_b)] \\
 \frac{1}{2} \tilde{f} \tilde{g}^* &= \frac{1}{2} AB^* \exp[i(k \cdot r - \omega t + \delta_a - k \cdot r + \omega t - \delta_b)] \\
 &= \frac{1}{2} AB^* \exp[i(\delta_a - \delta_b)]
 \end{aligned}$$

Thus,

$$\boxed{\text{Re} \left(\frac{1}{2} \tilde{f} \tilde{g}^* \right) = \frac{1}{2} AB \cos(\delta_a - \delta_b) = \langle fg \rangle}$$

#2. (Griffiths 9.18)

a) $\rho_r(t) = \exp(-\sigma/\epsilon)t \rho_r(0)$ (eq 9.120)

$$\tau_{\text{time to flow to the surface}} = \frac{\epsilon}{\sigma} \quad \epsilon = \epsilon_0 \epsilon_r, \quad \epsilon_r \approx n^2 \quad (\text{eq 9.70})$$

$$\approx \epsilon_0 n^2 / \sigma \quad \approx \epsilon_0 n^2 / (1/\rho) \quad n_{\text{glass}} = 1.5 \quad \text{average index of refraction for glass}$$

$$\approx 8.85 \times 10^{-12} \Omega^{-1} \cdot (1.5)^2 / (10^{-12} \Omega M) \leftarrow \text{from (Table 7.1)}$$

$$\boxed{\tau = 20 \text{ seconds}}$$

b) For silver: $\rho = 1.59 \times 10^{-8}$ (Table 7.1) $\epsilon \approx \epsilon_0$ $\omega = 10^{10} \text{ Hz} \cdot 2\pi$

$$\sigma = 1/\rho = 6.25 \times 10^7 \Rightarrow \omega\epsilon = 2\pi \times 10^{10} \times 8.85 \times 10^{-12} = .56$$

$$\text{Thus, eq (9.126)} \quad K = \sqrt{\frac{\epsilon\mu}{2}} \left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1 \right)^{1/2} \approx \sqrt{\frac{\omega\sigma\mu}{2}}$$

And thus, the skin depth is:

$$\text{eq (9.128)} \quad \delta = \frac{1}{K} \approx \sqrt{\frac{2}{\omega\sigma\mu}} = \sqrt{\frac{2}{2\pi \times 10^{10} \times 6.25 \times 10^7 \times 4\pi \times 10^{-7}}} = 6.4 \times 10^{-4} \text{ m}$$

Therefore the silver coating should be $\boxed{1.0 \times 10^{-6} \text{ m}}$

c) $\sigma_{\text{cu}} = 1/1.68 \times 10^{-8} = 6 \times 10^7$ (Table 7.1); $\omega\epsilon = (2\pi \times 10^6) \cdot (8.85 \times 10^{-12}) = 6 \times 10^{-5}$

$$\sigma \gg \omega\epsilon \quad \text{therefore, } K \approx \sqrt{\frac{\omega\sigma\mu}{2}} \quad \text{eq (9.126)}$$

And,

$$\lambda = 2\pi \sqrt{\frac{2}{\omega\sigma\mu_0}} = 2\pi \sqrt{\frac{2}{2\pi \times 10^6 \times 6 \times 10^7 \times 4\pi \times 10^{-7}}} = \boxed{4 \times 10^{-4} \text{ m}} \quad (\text{eq 9.129})$$

$$\text{in copper} \quad V = \lambda\nu = 4 \times 10^{-4} \times 10^6 = \boxed{400 \text{ m/s}}$$

$$\text{in vacuum} \quad \lambda = c/\nu = \frac{3 \times 10^8}{10^6} = \boxed{300 \text{ m}}$$

$$V = c = \boxed{3 \times 10^8 \text{ m/s}}$$

#3. (Griffiths 9.19)

a) $K = \omega \sqrt{\frac{\epsilon M}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\epsilon \omega} \right)^2} - 1 \right]^{1/2}$ (eq 9.126)

when $\omega \epsilon \gg \sigma$ we use the binomial expansion for the square root:

$$\approx \omega \sqrt{\frac{\epsilon M}{2}} \left(1 + \frac{1}{2} \left(\frac{\sigma}{\epsilon \omega} \right)^2 - 1 \right)^{1/2}$$

$$= \omega \sqrt{\frac{\epsilon M}{2}} \cdot \frac{1}{\sqrt{2}} \cdot \frac{\sigma}{\epsilon \omega} = \frac{\sigma}{2} \sqrt{\frac{M}{\epsilon}}$$

$$d = \boxed{\frac{1}{K} \approx \frac{2}{\sigma} \sqrt{\frac{\epsilon}{M}}} \quad (\text{eq 9.128})$$

For pure water, $\begin{cases} \epsilon = \epsilon_r \epsilon_0 = 80.1 \epsilon_0 & (\text{Table 4.2}) \\ M = M_0 (1 + \chi_M) = M_0 (1 - 9.0 \times 10^{-6}) \approx M_0 & (\text{Table 6.1}) \\ \sigma = 1 / 2.5 \times 10^5 & (\text{Table 7.1}) \end{cases}$

$$So, d \approx \frac{2}{1 / 2.5 \times 10^5} \sqrt{\frac{80.1 (8.85 \times 10^{-12})}{4\pi \times 10^{-7}}} = \boxed{1.19 \times 10^{-4} \text{ m}}$$

b) When $\sigma \gg \omega \epsilon$

$$K \approx k = \omega \sqrt{\frac{\epsilon M}{2}} \sqrt{\frac{\sigma}{\epsilon \omega}} = \sqrt{\frac{\omega M \sigma}{2}} \quad (\text{eq 9.126})$$

$$\lambda = 2\pi/k \approx 2\pi/\kappa = 2\pi d, \text{ or } \boxed{d = \frac{\lambda}{2\pi}}$$

$$d = \frac{1}{K} \approx \sqrt{\frac{2}{\omega M \sigma}} \quad \omega \approx 10^{15} \quad \epsilon \approx \epsilon_0 \quad M = M_0 \quad \sigma \approx 10^7 \text{ (RM)}^{-1}$$

$$= \sqrt{\frac{2}{10^{15} \cdot 4\pi \times 10^{-7} \cdot 10^7}} = \boxed{1/8 \times 10^{-7}} = \boxed{1.3 \times 10^{-8} \text{ m}}$$

• Therefore, light does not penetrate far into the metal - which accounts for its opacity.

c) From part (b) we had $k \approx K$

$$\text{Also, } \varphi = \tan^{-1}(K/k) \quad (\text{eq 9.134})$$

$$\text{Therefore } \boxed{\varphi = \tan^{-1}(1) = 45^\circ}$$

$$\frac{B_0}{E_0} = \sqrt{\epsilon\mu} \sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2} \approx \sqrt{\frac{\sigma\mu}{\omega}} \quad \sigma \gg \omega \epsilon \quad \text{eq (9.137)}$$

$$\text{For a typical metal: } \frac{B_0}{E_0} = \sqrt{\frac{10^3 \cdot 4\pi \times 10^{-7}}{10^{15}}} = \boxed{10^{-7} \text{ S/M}}$$

#4. (Griffiths 9.20)

$$\begin{aligned} a) \langle U \rangle &= \frac{1}{2} \left\langle \epsilon E^2 + \frac{1}{\mu} B^2 \right\rangle \\ &= \frac{1}{2} e^{-2kz} \left\langle \epsilon E_0^2 \cos^2(kz - \omega t + \delta_E) + \frac{1}{\mu} B_0^2 \cos^2(kz - \omega t + \delta_E + \varphi) \right\rangle \\ &= \frac{1}{2} e^{-2kz} \left(\frac{\epsilon}{2} E_0^2 + \frac{1}{2\mu} B_0^2 \right) \end{aligned} \quad \text{eq (9.138)}$$

$$= \frac{1}{4} e^{-2kz} \left(\epsilon E_0^2 + \frac{1}{\mu} E_0^2 \epsilon \mu \sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2} \right) \quad \text{eq (9.137)}$$

$$= \frac{1}{4} e^{-2kz} \epsilon E_0^2 \left(1 + \sqrt{1 + \left(\frac{\sigma}{\epsilon\omega}\right)^2} \right)$$

$$= \frac{1}{4} e^{-2kz} \epsilon E_0^2 \left(\frac{k^2}{\omega^2} \sqrt{\frac{2}{\epsilon\mu}} \right)^2 \quad (\text{eq 9.126})$$

$$= \frac{1}{4} e^{-2kz} \epsilon E_0^2 \left(\frac{k^2}{\omega^2} \frac{2}{\epsilon\mu} \right)$$

$$\boxed{\langle U \rangle = \frac{k^2}{2\mu\omega^2} E_0^2 e^{-2kz}}$$

$$\frac{\langle U_{\text{magnetic}} \rangle}{\langle U_{\text{electric}} \rangle} = \frac{B_0^2 / \mu}{E_0^2 \epsilon} = \frac{1}{\mu \epsilon} \frac{1}{\omega} \sqrt{1 + \left(\frac{\sigma}{\epsilon \omega}\right)^2} \quad (\text{eq 9.137})$$

$$\boxed{\frac{\langle U_{\text{mag}} \rangle}{\langle U_{\text{elec}} \rangle} = \sqrt{1 + \left(\frac{\sigma}{\epsilon \omega}\right)^2} > 1}$$

magnetic contribution
always dominates

b)

$$\begin{aligned} I &= \langle S \rangle = \langle \frac{1}{\mu} E \times B \rangle = \frac{1}{\mu} \langle E_0 B_0 e^{-2Kz} \cos(kz - \omega t + \delta_E) \\ &\quad \cos(kz - \omega t + \delta_E + \phi) \hat{z} \rangle \\ &= \frac{1}{2\mu} E_0 B_0 e^{-2Kz} \cos\phi \quad (\text{From problem #1}) \quad (\text{eq 9.138}) \\ &= \frac{1}{2\mu} E_0^2 e^{-2Kz} \left(\frac{K}{\omega} \cos\phi \right) \quad (\text{eq 9.135}) \end{aligned}$$

Also, $\frac{K}{k} = \tan\phi \implies \cos\phi = \frac{k}{(k^2 + K^2)^{1/2}} = \frac{k}{K}$

Thus, $\boxed{I = \frac{k}{2\mu} E_0^2 e^{-2Kz}}$

#5

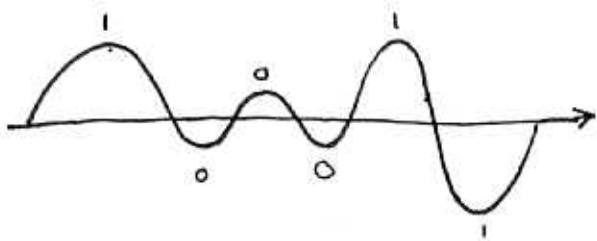
- a) In a deck there are 52 cards, to indicate which card he has picked he will need 6 bits.

$$\text{Since, } 52 \approx 2^6 = 64$$

Thus, Uri needs to send data at the speed of,

$$\frac{1 \text{ card}}{\text{second}} \cdot \frac{6 \text{ bits}}{\text{card}} = \boxed{6 \text{ bits/second}}$$

We will assume that the information is sent in a digital fashion (high/low peak) via an EM wave,



← this is a signal transmitting
110001 (6 bits of information)
second

The above signal has a frequency $\boxed{\nu = 3 \text{ Hz}}$

This roughly the minimum frequency Uri can use, however, to be sure about the accuracy of the data he should send a wave w/ twice the data capacity of GHz.

b) The skin depth equals:

$$\delta = \frac{1}{\kappa} = \frac{1}{\omega} \sqrt{\frac{2}{\epsilon \mu}} \left(\sqrt{1 + \left(\frac{\sigma}{\epsilon \omega} \right)^2} - 1 \right)^{1/2} \approx \frac{1}{\omega} \sqrt{\frac{2}{\epsilon \mu}} \sqrt{\frac{\epsilon \omega}{\sigma}} \quad \sigma \gg \epsilon \omega$$

$$= \sqrt{\frac{2}{\mu_0 \sigma \omega}}$$

To attenuate the EM waves generated by Uri's brain to $\approx \frac{1}{400} = e^{-6}$
we will need a shell with a width of six skin depths:

$$\boxed{\text{width of shell} = 6\delta}$$

We have the following physical constants for Al, Cu, and Fe:

	σ	μ	ρ_{density}	S/cm^3	
Al	3.77×10^7	M_\circ	2.699		$\mu_0 = 4\pi \times 10^{-7}$
Cu	5.95×10^7	M_\circ	8.96		$\omega = 18$
Fe	1.04×10^7	$500M_\circ$	7.874		

Using the above values

$$\boxed{\text{Al: } 6\delta = .28 \text{ m}}$$

$$\boxed{\text{Cu: } 6\delta = .23 \text{ m}}$$

$$\boxed{\text{Fe: } 6\delta = .024 \text{ m}}$$

In order to allow V_{ri} to have 1 m^3 of volume the shell has to have a radius equal to:

$$\frac{4}{3}\pi r_i^3 = 1\text{ m}^3 \Rightarrow r_i = .62\text{ m}$$

The mass of the different metal shells equals:

$$\text{mass} = \frac{4}{3}\pi((r_i + 6s)^3 - r_i^3) \rho_{\text{density}}$$

Al: mass = 5,500 kg

Cu: mass = 14,000 kg

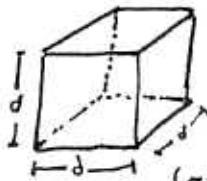
Fe: mass = 950 kg

Thus, we see that the ferromagnetic iron shell is the lightest.

Use Iron

#6

a)



$$\omega_0 = \frac{C}{d} \times \pi\sqrt{2} = \frac{3 \times 10^8}{.1} \times \pi\sqrt{2} = 1.3 \times 10^{10}$$

($\sigma \gg \omega$)
good conductor
• we get this result from problem #2

$$K \approx \sqrt{\frac{\omega \mu_0}{2}} = \sqrt{\frac{1.3 \times 10^{10} \cdot 4\pi \times 10^{-7} \cdot 6.3 \times 10^7}{2}} = \underline{\underline{7.3 \times 10^5 \text{ m}^{-1}}}$$

$$\text{Now, } Q \approx \frac{V}{AK^{-1}} = \frac{d^3}{6d^2 K^{-1}} = \frac{d}{6K^{-1}}$$

V : cavity's volume - d^3
 A : cavity's inside surface area - $6d^2$

$$Q \approx \frac{.1 \cdot 7.3 \times 10^5}{6} = 1.2 \times 10^4$$

b) $Q \approx \frac{dK}{6} = \frac{d}{6} \sqrt{\frac{\omega \mu_0}{2}} \Rightarrow$ Thus, we see that if ω goes up so will Q

Therefore, [increase ω to increase Q]

c) $Q \approx \frac{d}{6} \sqrt{\frac{\omega \mu_0}{2}}$ \Rightarrow Therefore, [yes it helps to increase Q if d is increased]

#7

$$R = \frac{\tilde{E}_{\text{out}}}{\tilde{E}_{\text{in}}} = \frac{1 - \tilde{\beta}}{1 + \tilde{\beta}} \quad (\text{eq 9.147})$$

$$\tilde{\beta} = \frac{\mu_1 V_1}{\mu_2 V_2} = \frac{\mu_0 V_1}{\mu_2 \omega / k_2} \quad (\text{eq 9.106})$$

and
(eq 9.146)

$$\boxed{\tilde{\beta} = \frac{Z_1}{Z_2}}$$

Thus,

$$R = \frac{1 - \tilde{\beta}}{1 + \tilde{\beta}} = \frac{1 - Z_1/Z_2}{1 + Z_1/Z_2} = \boxed{\frac{Z_2 - Z_1}{Z_2 + Z_1} = R}$$

$$T = \frac{\tilde{E}_{\text{out}}}{\tilde{E}_{\text{in}}} = \frac{2}{1 + \tilde{\beta}} \quad \text{eq (9.147)}$$

$$= \frac{2}{1 + Z_1/Z_2} = \boxed{\frac{2Z_2}{Z_2 + Z_1} = T}$$

#8

a) Given Information:

- $\mu_2 = \mu_0, \epsilon_2 = \epsilon_0, \sigma = \beta \epsilon_0 \omega$ where $\beta \ll 1$
- EM radiation from vacuum is normally incident upon this material

$$\frac{E_{\text{out}}}{E_{\text{in}}} = \frac{1 - \tilde{\beta}}{1 + \tilde{\beta}} \quad \tilde{\beta} = \frac{\mu_1 V_1}{\mu_2 \omega} \quad \tilde{k}_2 = \frac{\mu_1 V_1}{\mu_2 \omega} (k_2 + i\sigma) \quad (\text{eq 9.146})$$

$$k_2 = \omega \sqrt{\frac{\epsilon_0 \mu_0}{2}} \left(\sqrt{1 + \left(\frac{\sigma}{\epsilon_0 \omega} \right)^2} + 1 \right)^{1/2} = \omega \sqrt{\frac{\epsilon_0 \mu_0}{2}} \left(\sqrt{1 + \beta^2} + 1 \right)^{1/2} \quad (\text{eq 9.126})$$

$$\tilde{k}_2 = \omega \sqrt{\frac{\epsilon_0 \mu_0}{2}} (Z)^{1/2} = \omega \sqrt{\epsilon_0 \mu_0}$$

$$Z_1 = \frac{\tilde{E}_{\text{out}}}{H_{\text{in}}} = \frac{\tilde{E}_{\text{in}} e^{i(kz - \omega t)}}{\frac{1}{\mu_0} \tilde{E}_{\text{out}} e^{i(kz - \omega t)}} \quad \text{eq (9.148)}$$

$$Z_1 = \mu_0 V_1$$

$$Z_2 = \frac{\tilde{E}_{\text{out}}}{H_{\text{in}}} = \frac{\tilde{E}_{\text{in}} e^{i(kz - \omega t)}}{\frac{1}{\mu_2} \tilde{k}_2 \tilde{E}_{\text{out}} e^{i(kz - \omega t)}} \quad \text{eq (9.141)}$$

$$Z_2 = \mu_2 \omega / \tilde{k}_2$$

$$\begin{aligned} K_2 &= \omega \sqrt{\frac{\epsilon_2 \mu_0}{2}} \left(\sqrt{1 + \left(\frac{\sigma}{\epsilon_0 \omega}\right)^2} - 1 \right)^{1/2} \approx \omega \sqrt{\frac{\epsilon_0 \mu_0}{2}} \left(1 + \frac{1}{2} \beta^2 - 1 \right)^{1/2} \\ &= \omega \sqrt{\epsilon_0 \mu_0} \beta/2 \end{aligned}$$

Therefore,

$$\tilde{\beta} = \frac{\mu_0 V_i}{\mu_2 \omega} (k_2 + iK_2) = \frac{\mu_0 V_i}{\mu_0 \omega F} \omega \sqrt{\epsilon_0 \mu_0} (1 + i\beta/2) = \frac{1}{\sqrt{\epsilon_0 \mu_0}} (1 + i\beta/2)$$

$$\boxed{\tilde{\beta} = (1 + i\beta/2)}$$

$$\begin{aligned} \frac{\tilde{E}_{oR}}{\tilde{E}_{ox}} &= \frac{1 - \tilde{\beta}}{1 + \tilde{\beta}} = \frac{1 - 1 - i\beta/2}{1 + 1 + i\beta/2} = -i \frac{\beta/2}{2 + i\beta/2} \\ &\approx -i\beta/4 (1 - i\beta/4) = -i\beta/4 - \beta^2/16 \end{aligned}$$

$$\boxed{\tilde{E}_{oR}/\tilde{E}_{ox} \approx -i\beta/4}$$

the negative i indicates a phase lag of 90° of the reflected wave relative to the incident wave.

b) From part (a)

$$K = \omega \sqrt{\epsilon_0 \mu_0} \beta/2 = \frac{\omega}{c} \frac{\beta}{2} = \frac{2\pi\nu}{c} \frac{\beta}{2} = \frac{2\pi}{\lambda_0} \frac{\beta}{2}$$

So, $\boxed{\text{skin depth} = K^{-1} = \frac{\lambda_0}{\pi} \frac{1}{\beta}}$

$$\begin{aligned} \text{eq (9.134)} \quad \Phi_{\text{lag}} &= \tan^{-1} \left(\frac{K_2}{k_1} \right) = \tan^{-1} \left(\frac{\omega \sqrt{\epsilon_0 \mu_0} \beta/2}{\omega \sqrt{\epsilon_0 \mu_0}} \right) \\ &= \tan^{-1} (\beta/2) \quad \beta \ll 1 \end{aligned}$$

$$\boxed{\Phi_{\text{lag}} \approx \beta/2}$$

Therefore, H lags E by a phase shift equal to $\beta/2$.